Solutions

Exam 1
Chapter 1, 2 and 3.1-3.5
(without 3.4)

Multiple Choice
Answer the following multiple choice questions. Each question is worth 6 points. You may circle between 1 and 3 answers on each question. Circling 1 and getting the correct answer is worth 6 points. Circling 2 and getting the correct answer is worth 3 points. Circling 3 and getting the correct answer is worth 1 point.

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1. The expression $\frac{1}{2\sqrt[5]{x^2}}$ can be simplified in which of the following ways?

(a) $\frac{1}{2x^{-2/5}}$ (b) $(2x)^{-2/5}$
(c) $\frac{x^{-2/5}}{2}$ (d) $\frac{x^{2/5}}{2}$

2. Which of the following is the derivative of $\sin t \cos t$?

(a) $\cos^2 t - \sin^2 t$ (b) $-\sin t \cos t$
(c) $\sin t \cos t$ (d) 1

3. Find $\lim_{x\to 1^+} \frac{1-\sqrt{x}}{1-x}$.

(a) ∞ (b) $1/2$
(c) $-\infty$ (d) 0

4. On which interval does the equation $\tan x = x^2 - 1$ have a solution?

(a) $(-\pi/2, -\pi/4]$ (b) $[-\pi/4, 0]$
(c) $[0, \pi/4]$ (d) $[\pi/4, \pi/2)$

5. Find $\lim_{t\to\infty} t\tan(8t)$.

+6 pts Bad Problem.

(a) ∞ (b) 0
(c) 4 (d) 8

Should be $\lim_{t\to\infty} t \cdot \tan(8/t)$

6. Which of the following is the derivative of $\frac{\tan x}{\ln x}$?

(a) $\frac{\tan x/x - \ln x \sec^2 x}{\ln(x)^2}$ (b) $\frac{\ln x \sec^2 x - \tan x/x}{\ln(x)^2}$
(c) $\frac{\tan x/x - \ln x \sec^2 x}{\ln(x^2)}$ (d) $\frac{\ln x \sec^2 x - \tan x/x}{\ln(x^2)}$

7. Find the point (or points) where the function $f(x) = x^3 + 6x^2 - 36x + 100$ has horizontal tangent lines.

(a) $x = -2$ (b) there are no horizontal tangent lines
(c) $x = -6$ and $x = 2$ (d) $x = -2$ and $x = 2$

8. Suppose θ is an angle in the 3^{rd} or 4^{th} quadrant such that $\tan\theta = \sqrt{3}$. What is θ?

(a) $\frac{-\pi}{6}$ (b) $\frac{5\pi}{6}$
(c) $\frac{7\pi}{6}$ (d) $\frac{-2\pi}{3}$

Short Answer

Answer the following questions. *You must show your work to receive full credit.* Be sure to make reasonable simplifications. Indicate your final answer with a $\boxed{\text{box}}$.

1. (8 points) Find all asymptotes of the function $f(x) = \frac{2x^2-2}{5x^2-40x+35}$.

$$f(x) = \frac{2(x-1)(x+1)}{5(x-7)(x-1)} = \frac{2(x+1)}{5(x-7)} \quad \text{when } x \neq 1.$$

Horizontal Asymptotes: $\lim_{x\to\infty} f(x) = \frac{2}{5}$, $\lim_{x\to-\infty} f(x) = \frac{2}{5}$ $\left(y = \frac{2}{5}\right)$

Vertical Asymptotes: $\lim_{x\to 1} f(x) = \frac{-4}{30}$ no asymptote at $x=1$.

$\lim_{x\to 7^+} f(x) = \infty$ $\lim_{x\to 7^-} f(x) = -\infty$ $(x=7)$

2. (4 points) Use any method to find $\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$. Give at least one sentence of explanation. (Yes! That means words.)

$\lim_{h\to 0} \frac{e^{x+h}-e^x}{h}$ is the definition of the derivative of e^x. Therefore

$$\lim_{h\to 0} \frac{e^{x+h}-e^x}{h} = e^x.$$

3. (10 points) Find $\frac{dy}{dx}$, where

$$y = x^{\pi+1} - 5e^x + \ln x + 6\sin x - \frac{\cos x}{7}.$$

$$\frac{dy}{dx} = (\pi+1)x^{\pi} - 5e^x + \frac{1}{x} + 6\cos x + \frac{1}{7}\sin x.$$

4. (10 points) Find the tangent line to the function $g(x) = 6\cos(x)$ at the point $x = \pi/2$.

$$g'(x) = -6\sin x$$

Tangent Line: $y = g'(\pi/2)(x - \pi/2) + g(\pi/2)$

$$= -6(x - \pi/2) + 0$$

$$= -6x + 3\pi.$$

5. (10 points) For what values of a is the function

$$m(x) = \begin{cases} a^2x - 2a & x \geq 2 \\ 12x & x < 2 \end{cases}$$

continuous at every x?

Clearly $m(x)$ is continous on $(-\infty, 2)$ and $(2, \infty)$.

Only ~~to~~ need to check at $x=2$. So

$$\lim_{x \to 2^+} m(x) = \lim_{x \to 2^+} a^2x - 2a = 2a^2 - 2a = 2a(a-1)$$

$$\lim_{x \to 2^-} m(x) = \lim_{x \to 2^-} 12x = 24.$$

So $24 = 2a(a-1)$

$12 = a(a-1) = a^2 - a$

$0 = a^2 - a - 12 = (a-4)(a+3) \quad a = 4 \text{ or } -3$

6. Consider the function $h(x) = \frac{x^3+3x^2-10x}{x^2-4}$.

(a) (5 points) For what values of x is h discontinuous?

(b) (5 points) Create the function H which is a continuous extension of h at all points where this is possible.

(a) $h(x) = \frac{x(x+5)(x-2)}{(x+2)(x-2)} = \frac{x(x+5)}{x+2}$ when $x \neq 2$

discontinuous at $x = \pm 2$.

(b) Asymptote at $x=-2$. Best Continuous extension is

$$H(x) = \begin{cases} \frac{x^3+3x^2-10x}{x^2-4}, & x \neq \pm 2 \\ \lim_{x \to 2} h(x), & x = 2 \end{cases}$$

So $$H(x) = \begin{cases} \frac{x^3+3x^2-10x}{x^2-4}, & x \neq \pm 2 \\ 7/2, & x = 2 \end{cases}$$

Bonus Question

1. (8 points) Use the $\epsilon - \delta$ definition of limits to show that $f(x) = x^2 - 2$ is continuous at $x = 3$.

Let $\varepsilon > 0$. ~~Suppose~~ We expect $\lim_{x \to 3} f(x) = 7$.

So suppose $|f(x) - 7| < \varepsilon$

~~and~~

$$-\varepsilon < x^2 - 9 < \varepsilon$$

$$9 - \varepsilon < x^2 < \varepsilon + 9$$

$$\sqrt{9-\varepsilon} < x < \sqrt{\varepsilon + 9}$$

$$\sqrt{9-\varepsilon} - 3 < x - 3 < \sqrt{9+\varepsilon} - 3$$

Let $\delta = \min(\sqrt{9-\varepsilon} - 3, \sqrt{9+\varepsilon} - 3)$.

Then for $|x-3| < \delta$ we have $|f(x) - 7| < \varepsilon$.